OCR
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A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 2 hours

You must have:
- Printed Answer Booklet
- the Insert

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + \ldots + {}^nC_r\, a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad (|x|<1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}(x)}{\mathrm{f}'(x)}\,dx = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\left(\mathrm{f}(x)\right)^n dx = \frac{1}{n+1}\left(\mathrm{f}(x)\right)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small Angle Approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample Variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The Binomial Distribution

If $X \sim \mathrm{B}(n,\ p)$ then $P(X = r) = {}^n\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1 - p$

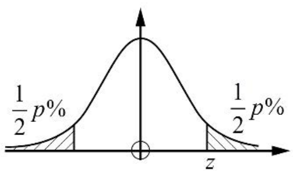
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (60 marks)

1 Express $\frac{2}{x-1}+\frac{5}{2x+1}$ as a single fraction. **[2]**

2 Find the first four terms of the binomial expansion of $(1-2x)^{\frac{1}{2}}$.

State the set of values of x for which the expansion is valid. **[4]**

3 Show that points A (1, 4, 9), B (0, 11, 17) and C (3, –10, –7) are collinear. **[4]**

4 Show that $\sum_{r=1}^{4} \ln \frac{r}{r+1} = -\ln 5$. **[3]**

5 **In this question you must show detailed reasoning.**

Fig. 5 shows the circle with equation $(x-4)^2+(y-1)^2=10$.
The points $(1, 0)$ and $(7, 0)$ lie on the circle. The point C is the centre of the circle.

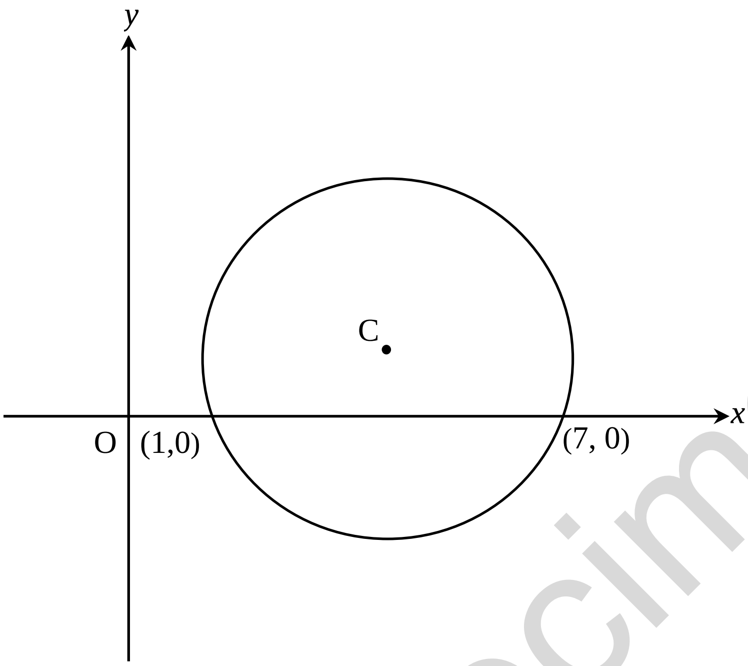


Fig. 5

Find the area of the part of the circle below the x-axis. **[5]**

6 **Fig. 6** shows the curve with equation $y = x^4 - 6x^2 + 4x + 5$.

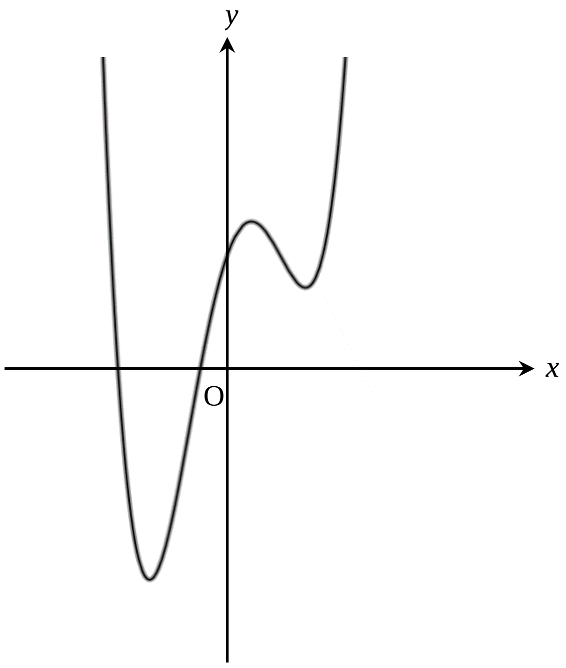


Fig. 6

Find the coordinates of the points of inflection. **[5]**

7 By finding a counter example, disprove the following statement.

If p and q are non-zero real numbers with $p < q$, then $\frac{1}{p} > \frac{1}{q}$. **[2]**

8 In **Fig. 8**, OAB is a thin bent rod, with OA = 1 m, AB = 2 m and angle OAB = 120°. Angles θ, ϕ and h are as shown in **Fig. 8**.

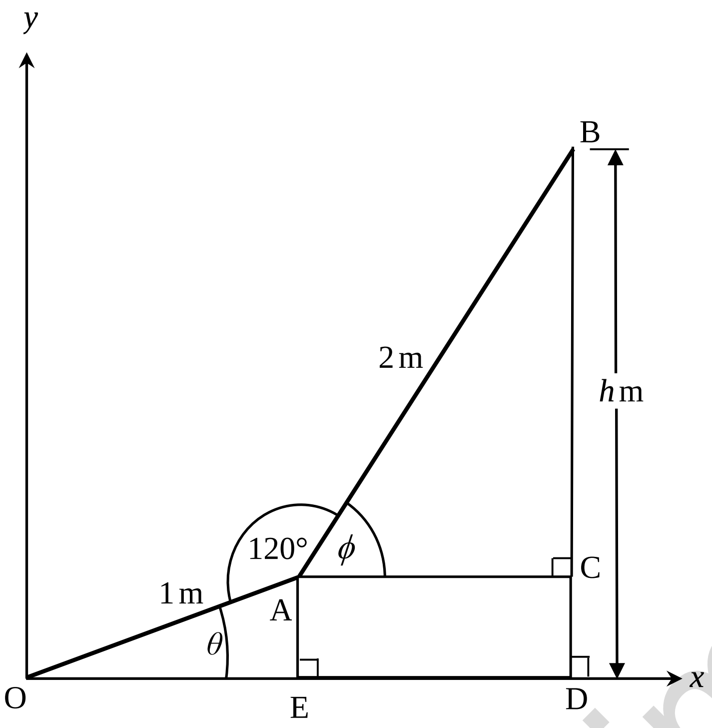


Fig. 8

(a) Show that $h = \sin\theta + 2\sin(\theta + 60°)$. **[3]**

The rod is free to rotate about the origin so that θ and ϕ vary. You may assume that the result for h in part **(a)** holds for all values of θ.

(b) Find an angle θ for which $h = 0$. **[5]**

9 **(a)** Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $0 < \alpha < \frac{1}{2}\pi$ and R is positive and given in exact form. **[4]**

The function $f(\theta)$ is defined by $f(\theta) = \dfrac{1}{(k + \cos\theta + 2\sin\theta)}$, $0 \le \theta \le 2\pi$, k is a constant.

(b) The maximum value of $f(\theta)$ is $\dfrac{(3+\sqrt{5})}{4}$.

Find the value of k. **[3]**

10 The function $f(x)$ is defined by $f(x) = x^4 + x^3 - 2x^2 - 4x - 2$.

(a) Show that $x = -1$ is a root of $f(x) = 0$. **[1]**

(b) Show that another root of $f(x) = 0$ lies between $x = 1$ and $x = 2$. **[2]**

(c) Show that $f(x) = (x+1)g(x)$, where $g(x) = x^3 + ax + b$ and a and b are integers to be determined. **[3]**

(d) Without further calculation, explain why $g(x) = 0$ has a root between $x = 1$ and $x = 2$. **[1]**

(e) Use the Newton-Raphson formula to show that an iteration formula for finding roots of $g(x) = 0$ may be written

$$x_{n+1} = \frac{2x_n^{\,3} + 2}{3x_n^{\,2} - 2}.$$

Determine the root of $g(x) = 0$ which lies between $x = 1$ and $x = 2$ correct to 4 significant figures. **[3]**

11 The curve $y = \mathrm{f}(x)$ is defined by the function $\mathrm{f}(x) = \mathrm{e}^{-x}\sin x$ with domain $0 \le x \le 4\pi$.

(a) **(i)** Show that the x-coordinates of the stationary points of the curve $y = \mathrm{f}(x)$, when arranged in increasing order, form an arithmetic sequence.

(ii) Show that the corresponding y-coordinates form a geometric sequence. **[9]**

(b) Would the result still hold with a larger domain? Give reasons for your answer. **[1]**

Answer **all** the questions

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

12 Explain why the smaller regular hexagon in **Fig. C1** has perimeter 6. **[1]**

13 Show that the larger regular hexagon in **Fig. C1** has perimeter $4\sqrt{3}$. **[3]**

14 Show that the two values of b given on line 36 are equivalent. **[3]**

15 **Fig. 15** shows a unit circle and the escribed regular polygon with 12 edges.

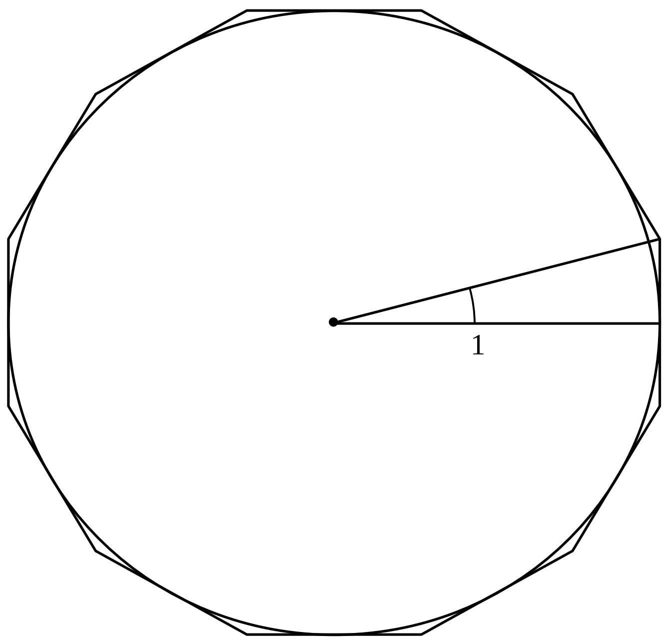


Fig. 15

(a) Show that the perimeter of the polygon is $24\tan 15°$. **[2]**

(b) Using the formula for $\tan(\theta - \phi)$ show that the perimeter of the polygon is $48 - 24\sqrt{3}$. **[3]**

16 On a unit circle, the inscribed regular polygon with 12 edges gives a lower bound for π, and the escribed regular polygon with 12 edges gives an upper bound for π.

Calculate the values of these bounds for π, giving your answers:

(i) in surd form

(ii) correct to 2 decimal places. **[3]**

END OF QUESTION PAPER